STATISTICS

Paper 4040/12
Paper 1

Key messages

Statistics is a practical subject, distinct from Mathematics, that is applied to a wide variety of real life situations. A candidate should read carefully, and understand, the situation described in a question in order to be able to produce the best possible answer.

A candidate should always be aware of whether or not the answer they have obtained to a question is possible and reasonable. If it is not, the work should be reviewed to find the error.

A question should always be read carefully so that the information that is given, and the task to be carried out, are both clearly understood and used properly.

General comments

The overall standard of work involving calculations of a routine nature was good. Some candidates also demonstrated sound understanding in interpreting the results of calculations. Others struggled to relate their findings to the practical situation in some questions.

It has been stressed repeatedly in these reports that a student of Statistics ought to realise when a numerical answer is reasonable for a particular practical situation. If it is obviously unreasonable, or even impossible, the work leading to it should be reviewed to find the mistake. Candidates need to appreciate that Statistics is not a form of pure mathematics devoid of practical relevance. For example, in finding the average length of an appointment at a dentist's surgery (see **Question 4** below) it should have been obvious that the answer could not possibly be several thousand minutes. In finding the number of passes in a subject at a school (see **Question 7** below) it should have been obvious that the number of passes could not possibly have been greater than the number of pupils taking the subject.

There was evidence on this paper of candidates either not reading carefully enough, or ignoring, information given in the question (see **Question 9** and **Question 11** below).

Comments on specific questions

Section A

Question 1

In **part (a)** candidates generally showed good awareness of the different sampling methods. In **part (b)**, whilst many were able to supply the term 'bias', fewer showed knowledge of the term 'representative'.

Answers: **(a)(i)** quota **(ii)** random **(iii)** systematic **(b)** bias, representative

Question 2

Part (i) was well done, with almost all candidates knowing how to calculate the basic measures of central tendency. In **part (ii)** the mode was the measure chosen most frequently, though the explanation offered was not always clear. The best answers pointed out that, for example, the three 28s in the rounded temperatures might actually have been recorded values of say, 27.9, 28.2 and 28.3, so that the original recordings would probably have contained no repeated value.

Answers: **(i)(a)** 27 **(b)** 28 **(c)** 26.3 **(ii)** mode; it is unlikely that for values recorded to one decimal place any would have occurred more than once

Question 3

Performance on the Venn diagram question was again very varied. There were only a few full mark answers, and a good number earning scarcely any marks. Many candidates still demonstrate limited understanding of what the different regions of a Venn diagram represent.

Answers: **(i)** 10 **(ii)** 5 **(iii)** 26 **(iv)** 9 **(v)** 6

Question 4

Almost all candidates showed by their answers to **part (i)** that the appropriate column of measures given in the table had been consulted.

There were far fewer fully correct answers to **parts (ii)** and **(iii)**. Not only were there so many incorrect answers to these two questions, but, even more seriously, many of the answers offered were totally unrealistic. Thus it was quite common to see the surgery holding a non-integer number of appointments in a week, and further, the afternoon appointments lasting on average thousands of minutes. As is remarked regularly in these reports candidates ought to be aware of whether or not the answer they are presenting is a reasonable one for the practical situation of the question.

Answers: **(i)(a)** morning **(b)** afternoon **(ii)** 85 **(iii)** 45 minutes

Question 5

Reasonable understanding of the two-way table was shown. Many candidates obtained the marks in **part (i)**, but a fairly common incorrect answer to **part (ii)** was 22, the number of matches played. Only a few candidates gained the mark in **part (iii)**. Almost all seemed to think that it was enough to point out that the table contains a 5 or more column for goals scored. Yet the fact there is a 5 or more column also for goals conceded had not prevented the calculation of the number of goals conceded by the team in the season in **part (ii)**. So for a fully acceptable answer it was necessary to say also that there were matches the team had played (four to be precise) in which 5 or more goals were scored (the team had played no matches in which 5 or more goals were conceded).

Answers: **(i)(a)** 10 **(b)** 9 **(c)** 6 **(ii)** 55 **(iii)** there were four matches played in each of which we know only that 5 or more goals were scored, so we cannot calculate the goals scored in these matches

Question 6

This question tended to be either well done or poorly done. The most common error in otherwise good answers was the omission of the second possibility in **part (i)(c)**. Another common error was to treat the situation as though exactly 100 essays had been submitted. Some candidates offered impossible answers of fewer than 134 in **part (ii)**.

Answers: **(i)(a)** 0.8464 **(b)** 0.0736 **(c)** 0.003 **(ii)** 1675

Section B

Question 7

In recent years crude and standardised rates have been applied to a range of practical situations, and generally candidates continue to adapt their knowledge well to these situations. For example, in this question there was scarcely any mention of ‘deaths’ or the use of 1000.

At the start of the question there was much greater success in finding the standardised rate in **part (i)** than the crude rate in **part (ii)**. Some answers to **part (ii)** provided another instance of totally impossible numbers being used in calculations. Many laboured through percentage working to find the number of passes in each ability group ending, for example, with 8 passes in the modest ability group out of the 4 such pupils enrolled.

In **parts (iii)** and **(iv)** the correct choice was almost always made, but in **part (iv)** the necessary reference to number of enrolments was very rarely seen.

Candidates who had been successful in **part (ii)** usually achieved success also in **part (v)**. The very elementary error of averaging the three crude rates was usually avoided.

Answers: **(i)** 84% **(ii)** 82.6% **(iii)** Japanese, because it has the highest standardised pass rate
(iv) Chemistry, because it has the highest crude pass rate and the highest number of enrolments
(v) 75.9%

Question 8

There were many fully correct answers to **part (i)**, but few to the related **part (iii)**. Good answers to **part (iii)** showed clear understanding that, after completion of the new building, not only would it be necessary to add the height of this building to the old total height, but also to remove the height of one of the smallest buildings from the old total. The least creditworthy answers simply added in the new height, or formed a new unnecessary class including the new height, and divided by 26, even though the mean for the 25 tallest was required.

The histograms seen in **part (ii)** fell into two categories: those that showed clear understanding of the principle on which a histogram is constructed, and those drawn simply with column heights equal to the given frequencies.

The strongest answers to the probability parts of the question indicated clearly that these are all ‘without replacement’ situations, and that there are three cases to consider in **part (v)** and six in **part (vi)**. The most limited answers showed no appreciation of the fact that a product of three probabilities is needed in each part, corresponding to the three choices made for safety checks.

Answers: **(i)** 104.4 m **(ii)** column heights 6, 8, 3.5, 2 **(iii)** 107.6 m **(iv)** $\frac{7}{228}$ **(v)** $\frac{9}{190}$ **(vi)** $\frac{21}{95}$

Question 9

Part (i) was well answered with the best answers showing very clearly, with lines drawn on the graph, how results for the different parts had been found. **Part (ii)(a)** was answered correctly by almost all candidates, but a common error in **part (ii)(b)** was to read from a cumulative frequency of 28 instead of 36. A much more serious error, resulting in substantial mark loss, was to work with a total frequency of 70 throughout the whole question. The fact that the total frequency of 64 was stated in the first sentence of the question, and also clearly shown on the graph, seems to have been overlooked.

Good understanding of the probability situation was shown in **part (iii)** by candidates who recognised that the product of four ‘without replacement’ probabilities was required. But few of those successful in doing this saw the link between **parts (iii)** and **(iv)**. Long direct attempts at **part (iv)** were seen which were almost always unsuccessful because of the very large number of cases involved. Many candidates also seemed to think that **part (iv)** required the probability of only one type of bread meeting the government’s recommendation.

In **part (v)** the correct choice was usually made, but not always properly explained.

Answers: **(i)(a)** 1.16 **(b)** 0.17 **(c)** 93.75 **(ii)(a)** 8 **(b)** 1.18 **(iii)** 0.578 **(iv)** 0.422
(v) interquartile range; both the lower and upper quartiles will be smaller by 0.05, so the difference between them will remain unchanged

Question 10

Answers to the first three parts were generally very good, with clearly plotted points, a correctly calculated lower semi-average, and a well-explained method in finding the line of best fit. Most candidates used the averages in finding the latter, but a few used instead points chosen from their line. As is pointed out regularly in these reports, candidates need to be aware that, whilst it is perfectly correct to choose points from the line, it commonly results in loss of accuracy. Values read from the line are almost always less precise than the calculated averages, through which the line must necessarily pass. Scarcely any made the elementary error of using data points to find the equation of the line (though this can occasionally be valid if the line happens to pass through one or more).

The line was well used in **part (iv)**, the best answers indicating with further lines drawn on the graph how the prices were obtained. Only a few confused asking price and selling price, but many did not express answers to the nearest $5, as the question directed. **Part (v)** proved the most challenging overall, with few candidates able to see that, from the equation of the line, the actual selling price was (with correct earlier calculation) approximately 74% of the asking price.

The appropriate choice and reason were commonly given in answers to **part (vi)**. No credit was allowed for answers involving profit as no information is given in the question about what the trader paid for any of the items in acquiring them.

Answers: **(ii)** (21.25, 15) **(iii)** $y = 0.742x - 0.766$ **(iv)(a)** $40 **(b)** $70 **(v)** approximately 26(%)
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Question 11

The pictogram was very clearly understood, and many fully correct answers to the first three parts were seen.

The strongest answers to **part (iv)** took account of the different numbers of males and females in the survey, calculated the radius of the chart to be drawn, and produced a diagram with accurate radius, angles, and full labelling. Far too many candidates ignored totally the fact that fewer females than males were surveyed, and drew a second chart with much the same radius as the one given.

Good answers to **part (v)** expressed all three conclusions in terms of proportions. Answers given in terms of absolute numbers could not be credited in those cases where the different numbers of males and females had been ignored in constructing the second chart, and the conclusions were to be drawn from comparing the charts. The answers given below are not exhaustive; others were accepted if appropriate, provided the chart for females was drawn smaller than the chart for males.

In **part (vi)** the best answers showed sound understanding of the essential distinction of purpose between representing survey data in one of these two ways. Vague, insubstantial responses such as ‘easy to draw’, ‘easy to understand’, ‘less time consuming’, were not accepted.

In **part (vii)** it was necessary to relate the answer to the particular survey conducted in the question. Some candidates offered general observations on the nature of open and closed questions.

Answers: **(i)** 33 **(ii)** 8 **(iii)** 80% **(iv)** chart drawn with radius 3 cm, angles of 192°, 96°, 72°, fully labelled and correct **(v)** larger proportion of females than males is in favour of the ban; a smaller proportion of females than males is against the ban; the proportions of females and males undecided are the same **(vi)(a)** clear visual representation of numbers in categories **(b)** clear visual representation of relative proportions in categories **(vii)** closed; customers seem to have been restricted to giving one of only three possible responses

STATISTICS

Paper 4040/13
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STATISTICS

Paper 4040/22
Paper 2

Key message

This examination requires candidates to be able to produce statistical diagrams, to calculate statistics and to interpret findings. The best statistical diagrams should be accurately drawn, taking care to use scales correctly, and have clearly labelled axes and, where appropriate, a key. Candidates scoring the highest marks in the numerical problems will provide clear indications of the methods they have used in logical and clearly presented solutions. In questions that required written definitions, justification of given techniques or interpretation, the most successful responses will include detailed explanations with, where appropriate, clear consideration of the context of the problem.

General comments

It was pleasing to see improved accuracy in the drawing of the chart in **Question 1** and the plotting of the graphs in **Question 3** and **Question 11**. In particular there was an improvement from previous years in the labelling of the chart and graph in **Question 1** and **Question 3**. As usual, candidates did better on the questions requiring numerical calculations than on those requiring written explanations; in particular, candidates did well in parts of the two probability questions in ***Section A***, namely **Question 4** and **Question 6**, and in the numerical parts of **Question 8** on price relatives and **Question 11** on moving averages. It was particularly pleasing to see so many correct solutions to **Question 6(ii)** and clear explanations of the result in **Question 9(i)**. Where a question required the interpretation of a diagram or a statistic, these interpretations were sometimes incorrect, such as in **Question 3(ii)**, and sometimes insufficiently detailed, such as in **Questions 5(ii)**, **8(iii)(b)** and **10(i)**. It was pleasing, however, to see so many candidates providing appropriate context to their interpretations, such as in **Question 8(iv)**.

Question 9, on probability and expectation, proved to be the least popular of the optional ***Section B*** questions, with **Question 8**, on price relatives and index numbers and **Question 11**, on moving averages, being the most popular.

Comments on specific questions

Section A

Question 1

In **part (i)**, the vast majority of candidates interpreted the dual bar chart correctly to find the total number of male and female students. It was pleasing to see so many candidates then make the correct percentage calculations for the percentage sectional bar chart in **part (ii)**. These were generally accurately drawn using the given scale, with clear keys and labelling provided by most candidates.

Answers: **(i)** 39, 34; **(ii)** 31, 41, 28 and 56, 12, 32.

Question 2

Most candidates correctly identified ‘the height of each competitor’ as a continuous quantitative variable and ‘number of events entered’ as a discrete quantitative variable in **part (i)**. The qualitative variables listed, namely ‘country of origin of each competitor’ and ‘name of each competitor’, were not always identified as such; in particular many candidates considered the ‘name of each competitor’ not to be a variable. ‘The winning time for the men’s 110 m hurdles’ was often incorrectly identified as a continuous variable, rather than not a variable at all.

In **parts (ii)** and **(iii)** there was a large difference between the number of candidates able to find correct boundaries for the ages and the masses. Most candidates could correctly give the boundaries for the 50–54 class representing masses measured to the nearest kg, in **part (iii)**, but very few gave correct lower and upper class boundaries for the 19–21 class representing ages in completed years in **part (ii)**. Often candidates treated this class in the same ways as the masses, giving 18.5 and 21.5 as their answers or they gave a correct lower bound, but an upper bound of 21.

Answers: **(ii)** 19, 22; **(iii)** 49.5, 54.5.

Question 3

Most candidates were able to attempt to draw a pair of frequency polygons in **part (i)**, although a few candidates drew cumulative frequency curves instead. A few histograms were also seen, some of which had frequency polygons correctly superimposed and identified by a key. In this question it was necessary for the candidates to provide their own linear scales, which almost all did successfully. It was very pleasing to see that these scales were usually correctly labelled, including the units on the axis illustrating height. Keys or labelling of the polygons was also usually present to indicate which polygon was for male and which for female elephants. A common error was for plots to be made at the upper boundaries rather than the mid-points of each class.

Many found the interpretation of the polygons in **part (ii)** much more difficult. It was common to see the female elephants, rather than the male elephants, being described as having the generally greater shoulder heights. It was not sufficient simply to compare the numbers of male and female elephants within one of the height classes; rather it was necessary to make a general comparison based on the data from the full range of heights.

Answers: **(ii)** Male elephants have generally greater shoulder heights.

Question 4

In **part (a)(i)** it was pleasing to see that most candidates could use the probability formula for independent events, $P(A \cap B) = P(A) \cdot P(B)$, to find $P(B)$. In **part (a)(ii)**, however, many were not able to describe a possible example for such an event. Many candidates suggested 'obtaining a tail when the fair coin is thrown'. This event has the correct probability of 0.5, but is not independent of event A. The most commonly seen correct answers were 'obtaining a tail when the coin is thrown **again**', 'obtaining a head on a **different** coin' or 'obtaining an even number when a fair die is thrown'.

Many candidates used the probability formula for mutually exclusive events, $P(C \cup D) = P(C) + P(D)$, to obtain the correct answer to the first question in **part (b)**. Some, however, reverted to the formula for independent events when trying to find $P(C \cap D)$.

Answers: **(a)(i)** 0.5; **(b)** 0.83, 0.

Question 5

Most candidates seemed to know what was required for a systematic sample, in **part (i)**, listing houses at equal intervals, although some did not realise that the interval size should be 8. A few candidates gave a simple random rather than a systematic sample.

In **part (ii)(a)** most candidates correctly explained that their sample consisted of houses from one side of the road only. Many candidates, however, did not comment on the fact that the adults in just five households were to be surveyed and that all the adults in any one particular house may well have similar opinions. To obtain a more representative sample in **part (ii)(b)**, many candidates suggested an appropriate method, such as a sample stratified by side of the road, but very few said that this should be a sample of the people rather than of the houses.

Answers: **(i)** 02, 10, 18, 26, 34.

Question 6

This question on probability was generally well done. Correct answers were usually seen in **parts (i)(a)** and **(i)(b)**, with just a few candidates multiplying rather than adding the probabilities in **part (i)(b)**. An incorrect denominator of 25, rather than 15, was seen quite often in **part (i)(c)**. A common error seen in **part (i)(d)** was for the probability of the white short-sleeved shirts to be added to rather than subtracted from the sum of P(white) + P(short-sleeved).

It was **part (ii)** where a surprising number of fully correct solutions were seen. A small number of candidates incorrectly assumed replacement in this situation.

Answers: **(i)(a)** 4/25; **(i)(b)** 19/25; **(i)(c)** 2/15; **(i)(d)** 17/25; **(ii)** 8/25.

Section B

Question 7

The earlier parts of this question proved more difficult for some candidates than the later parts. In **part (i)** a common incorrect answer seen was 80 000.

In **part (ii)(a)** most candidates correctly gave the median as the most appropriate measure of central tendency to represent the data, although it was surprisingly common to see the mode, and a few candidates gave answers that are not measures of central tendency such as the interquartile range. Common incorrect answers seen in **part (ii)(b)** were 80 000, 10 000, and values from the frequencies such as 33, 29 or 0.

Part (iii) was often well done with many candidates gaining full marks. A fairly small minority decided at the outset to subtract 30 from 90 and therefore incorrectly found the 60th value.

Many good attempts at **part (iv)** were seen. Some candidates began well but stopped, having used linear interpolation to find an estimate for the number of cars that had completed less than 22 000 km. It is important, at the end of a multistage problem, to re-read the question to make sure that it has been answered fully.

Answers: **(i)** 70 000; **(ii)(a)** Median; **(ii)(b)** Any values between 60 000 and 80 000 (but not including 80 000); **(iii)** 10 500; **(iv)** 5060.

Question 8

Some excellent work was seen in the first three parts of this question. In **part (i)** most candidates correctly found the expenditure for each category and attempted to reduce this to a ratio in its lowest terms. A few arithmetic errors were made and a few candidates used just the amounts rather than the expenditure when giving the ratio. Correct calculations to find the price relatives were usually seen in **part (ii)**. Quite a large number of candidates unnecessarily calculated the new cost of leaflets by increasing the cost of them last year by 3% and then calculated that this increase was indeed 3%. The most common error was in the calculation for phone calls, with some candidates increasing rather than decreasing the cost. The vast majority of candidates knew how to find the weighted aggregate cost index in **part (iii)(a)**, with most correctly following the instruction to give the answer correct to one decimal place.

The interpretation of the weighted aggregate cost index, in **part (iii)(b)**, was often missing some detail. The three parts required for this answer are that the figure shows a decrease in the overall cost, that the amount of this decrease is 1.6% and that this has occurred over the time period of one year. The aspect most often missing was the time period, followed by 1.6% which was sometimes missing the percent sign.

In **part (iv)** most candidates provided reasons that were correctly connected to possible changes in the amounts; namely changes to the number of leaflets, the length of phone calls or the amount of petrol. A few candidates incorrectly made reference to possible changes to prices. It was pleasingly rare to see an answer lacking context, such as simply ‘the amounts may change’.

Answers: **(i)** 6 : 10 : 1; **(ii)** 103, 95, 105; **(iii)(a)** 98.4.

Question 9

Some very well explained solutions were seen to show the given result in **part (i)**. Calculations of the expected earnings in **part (ii)** were usually either fully or partially correct, with most candidates demonstrating a clear understanding of what is meant by expectation. Two alternative approaches were seen; either ‘16.50 · 0.78 + 14.50 · 0.22’ or ‘16.50 – 2 · 0.22’. The results from the first two parts of this question were successfully used by most candidates to answer **part (iii)**.

Part (iv) proved to be the most difficult part of this question. The most common approaches seen were ‘*y* · 0.78 + (*y* – 3) · 0.22 = 16.06’, ‘*y* · 39 +(*y* – 3) · 11 = 803’ or ‘*y* – 3 · 0.22 = 16.06’. Some candidates who attempted the first two of these omitted essential brackets in their solution. Fully correct solutions were often seen in **part (v)** and most solutions contained at least one correct product.

Answers: **(i)** 0.8 · 0.1 + 0.2 · 0.7 = 0.22; **(ii)** $16.06; **(iii)** 11, $803; **(iv)** $16.72 **(v)** 0.3448.

Question 10

Most candidates were able to score at least one mark in **part (i)**. Comparing the means, most candidates were able to state, for example, that times to complete the first circuit tended to be quicker. It was not sufficient simply to state that the mean for the first circuit was smaller; some interpretation of this in terms of the times taken was needed. Fewer candidates successfully compared the standard deviations. For the second mark it was necessary to state, for example, that the times for the first circuit were less varied. Some candidates, having gained the first mark by saying that the people ran faster in the first circuit, gave, as their second difference, the equivalent statement, i.e. that the people ran slower in the second circuit, which did not gain a second mark.

In **part (ii)** most candidates were able to calculate the scaled times, but they were not always able to interpret these correctly. It was necessary to understand that a lower scaled time suggested a better performance relative the rest of the people.

Most candidates were able to find the mean and the standard deviation in **part (iii)(a)**, but many were not able to do so by using the assumed mean. Many scripts included abandoned attempts involving the assumed mean. For those that persisted and who correctly found mid-points and subtracted the assumed mean, the multiplication by the frequency was often forgotten. Those that successfully found the mean using the assumed mean often abandoned its use for the calculation of the standard deviation.

Many candidates were able to explain in **part (iii)(b)** that these answers were estimates because the data was grouped or because mid-points had been used. A few thought incorrectly that they were estimates because of the use of the assumed mean. Some candidates left **part (iii)(c)** blank, but there were some good suggestions, such as increasing the number of classes or providing the raw data. Some candidates even provided suggestions for some specific new class intervals.

Answers: **(ii)** 1.5, 1.25; **(iii)(a)** 265.6, 22.6.

Question 11

There were some very good solutions seen to the numerical parts of this question, but the explanations were found to be more difficult. In **part (i)** most candidates could explain that 5 was a suitable value for *n*; the most common explanation seen was that there are 5 days in this school week, others clearly explained that the pattern seen in the data repeats every 5 days. To explain why centring is not necessary, many candidates provided the simple explanation that *n* is odd. However, for full marks it was necessary to explain further that, as a consequence, the moving average values will coincide with the original data. This more detailed explanation was often missing. The majority of candidates correctly found all six 5-point moving average values and most correctly inserted them from the Wednesday of week 1 to the Wednesday of week 2 in the table. A few candidates inserted them in the wrong place and some only calculated the first and the last of these values or only calculated five of them.

In **part (ii)** the most able candidates provided fully correct solutions. Some found the two correct differences between the appropriate moving average values and the number of meals, but did not then find the mean of these. Incorrect solutions often involved finding the difference between the two Wednesday moving average values or finding the mean of the two Wednesday moving average values without looking at the difference between these values and the number of meals.

Plots in **part (iii)** tended to be accurate with suitable trend lines drawn. A small number of candidates incorrectly simply joined their plotted points.

As in previous years, candidates found **part (iv)** the most difficult of the numerical parts of this question, with a common error being simply to take a reading from the trend line of the graph and not to make use of the seasonal component they had calculated in **part (ii)**.

In **part (v)** most candidates correctly stated that the trend in the number of meals consumed was increasing. Many, however, found it difficult to explain whether or not they felt it was reasonable to assume that this trend would continue, with many simply stating that they felt it either was or was not reasonable without attempting an explanation. The best explanations considered the fact that there was likely to be a point at which the number of meals consumed reach a maximum value due to the limit of the number of pupils in the school, and therefore the trend would not continue over the long term.

Answers: **(i)(c)** 253.8, 255, 256, 257.6, 259.4, 261; **(ii)** 27.6; **(iv)** e.g. 295 or 296; **(v)(a)** increasing.

STATISTICS

Paper 4040/23
Paper 2

Key message

This examination requires candidates to be able to produce statistical diagrams, to calculate statistics and to interpret findings. The best statistical diagrams should be accurately drawn, taking care to use scales correctly, and have clearly labelled axes and, where appropriate, a key. Candidates scoring the highest marks in the numerical problems will provide clear indications of the methods they have used in logical and clearly presented solutions. In questions that required written definitions, justification of given techniques or interpretation, the most successful responses will include detailed explanations with, where appropriate, clear consideration of the context of the problem.

General comments

It was pleasing to see improved accuracy in the drawing of the chart in **Question 1** and the plotting of the graphs in **Question 3** and **Question 11**. In particular there was an improvement from previous years in the labelling of the chart and graph in **Question 1** and **Question 3**. As usual, candidates did better on the questions requiring numerical calculations than on those requiring written explanations; in particular, candidates did well in parts of the two probability questions in ***Section A***, namely **Question 4** and **Question 6**, and in the numerical parts of **Question 8** on price relatives and **Question 11** on moving averages. It was particularly pleasing to see so many correct solutions to **Question 6(ii)** and clear explanations of the result in **Question 9(i)**. Where a question required the interpretation of a diagram or a statistic, these interpretations were sometimes incorrect, such as in **Question 3(ii)**, and sometimes insufficiently detailed, such as in **Questions 5(ii)**, **8(iii)(b)** and **10(i)**. It was pleasing, however, to see so many candidates providing appropriate context to their interpretations, such as in **Question 8(iv)**.

Question 9, on probability and expectation, proved to be the least popular of the optional ***Section B*** questions, with **Question 8**, on price relatives and index numbers and **Question 11**, on moving averages, being the most popular.

Comments on specific questions

Section A

Question 1

In **part (i)**, the vast majority of candidates interpreted the dual bar chart correctly to find the total number of male and female students. It was pleasing to see so many candidates then make the correct percentage calculations for the percentage sectional bar chart in **part (ii)**. These were generally accurately drawn using the given scale, with clear keys and labelling provided by most candidates.

Answers: **(i)** 39, 34; **(ii)** 31, 41, 28 and 56, 12, 32.

Question 2

Most candidates correctly identified ‘the height of each competitor’ as a continuous quantitative variable and ‘number of events entered’ as a discrete quantitative variable in **part (i)**. The qualitative variables listed, namely ‘country of origin of each competitor’ and ‘name of each competitor’, were not always identified as such; in particular many candidates considered the ‘name of each competitor’ not to be a variable. ‘The winning time for the men’s 110 m hurdles’ was often incorrectly identified as a continuous variable, rather than not a variable at all.

In **parts (ii)** and **(iii)** there was a large difference between the number of candidates able to find correct boundaries for the ages and the masses. Most candidates could correctly give the boundaries for the 50–54 class representing masses measured to the nearest kg, in **part (iii)**, but very few gave correct lower and upper class boundaries for the 19–21 class representing ages in completed years in **part (ii)**. Often candidates treated this class in the same ways as the masses, giving 18.5 and 21.5 as their answers or they gave a correct lower bound, but an upper bound of 21.

Answers: **(ii)** 19, 22; **(iii)** 49.5, 54.5.

Question 3

Most candidates were able to attempt to draw a pair of frequency polygons in **part (i)**, although a few candidates drew cumulative frequency curves instead. A few histograms were also seen, some of which had frequency polygons correctly superimposed and identified by a key. In this question it was necessary for the candidates to provide their own linear scales, which almost all did successfully. It was very pleasing to see that these scales were usually correctly labelled, including the units on the axis illustrating height. Keys or labelling of the polygons was also usually present to indicate which polygon was for male and which for female elephants. A common error was for plots to be made at the upper boundaries rather than the mid-points of each class.

Many found the interpretation of the polygons in **part (ii)** much more difficult. It was common to see the female elephants, rather than the male elephants, being described as having the generally greater shoulder heights. It was not sufficient simply to compare the numbers of male and female elephants within one of the height classes; rather it was necessary to make a general comparison based on the data from the full range of heights.

Answers: **(ii)** Male elephants have generally greater shoulder heights.

Question 4

In **part (a)(i)** it was pleasing to see that most candidates could use the probability formula for independent events, $P(A \cap B) = P(A) \cdot P(B)$, to find $P(B)$. In **part (a)(ii)**, however, many were not able to describe a possible example for such an event. Many candidates suggested 'obtaining a tail when the fair coin is thrown'. This event has the correct probability of 0.5, but is not independent of event A. The most commonly seen correct answers were 'obtaining a tail when the coin is thrown **again**', 'obtaining a head on a **different** coin' or 'obtaining an even number when a fair die is thrown'.

Many candidates used the probability formula for mutually exclusive events, $P(C \cup D) = P(C) + P(D)$, to obtain the correct answer to the first question in **part (b)**. Some, however, reverted to the formula for independent events when trying to find $P(C \cap D)$.

Answers: **(a)(i)** 0.5; **(b)** 0.83, 0.

Question 5

Most candidates seemed to know what was required for a systematic sample, in **part (i)**, listing houses at equal intervals, although some did not realise that the interval size should be 8. A few candidates gave a simple random rather than a systematic sample.

In **part (ii)(a)** most candidates correctly explained that their sample consisted of houses from one side of the road only. Many candidates, however, did not comment on the fact that the adults in just five households were to be surveyed and that all the adults in any one particular house may well have similar opinions. To obtain a more representative sample in **part (ii)(b)**, many candidates suggested an appropriate method, such as a sample stratified by side of the road, but very few said that this should be a sample of the people rather than of the houses.

Answers: **(i)** 02, 10, 18, 26, 34.

Question 6

This question on probability was generally well done. Correct answers were usually seen in **parts (i)(a)** and **(i)(b)**, with just a few candidates multiplying rather than adding the probabilities in **part (i)(b)**. An incorrect denominator of 25, rather than 15, was seen quite often in **part (i)(c)**. A common error seen in **part (i)(d)** was for the probability of the white short-sleeved shirts to be added to rather than subtracted from the sum of P(white) + P(short-sleeved).

It was **part (ii)** where a surprising number of fully correct solutions were seen. A small number of candidates incorrectly assumed replacement in this situation.

Answers: **(i)(a)** 4/25; **(i)(b)** 19/25; **(i)(c)** 2/15; **(i)(d)** 17/25; **(ii)** 8/25.

Section B

Question 7

The earlier parts of this question proved more difficult for some candidates than the later parts. In **part (i)** a common incorrect answer seen was 80 000.

In **part (ii)(a)** most candidates correctly gave the median as the most appropriate measure of central tendency to represent the data, although it was surprisingly common to see the mode, and a few candidates gave answers that are not measures of central tendency such as the interquartile range. Common incorrect answers seen in **part (ii)(b)** were 80 000, 10 000, and values from the frequencies such as 33, 29 or 0.

Part (iii) was often well done with many candidates gaining full marks. A fairly small minority decided at the outset to subtract 30 from 90 and therefore incorrectly found the 60th value.

Many good attempts at **part (iv)** were seen. Some candidates began well but stopped, having used linear interpolation to find an estimate for the number of cars that had completed less than 22 000 km. It is important, at the end of a multistage problem, to re-read the question to make sure that it has been answered fully.

Answers: **(i)** 70 000; **(ii)(a)** Median; **(ii)(b)** Any values between 60 000 and 80 000 (but not including 80 000); **(iii)** 10 500; **(iv)** 5060.

Question 8

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